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Influence of Double Dispersion on Natural Convection Flow over a Vertical Cone Saturated Porous Media with Soret and Dufour Effects

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ABSTRACT
The proposed work in this manuscript reflects the influence of double dispersion in the presence of Soret and Dufour effects due to natural convection flow over a vertical cone saturated porous medium. The non-dimensionalization performed through similarity variables and the attained similarity equations are solved numerically via the Chebyshev spectral quasi-linearization method. The impression of governing parameters on the flow characteristics is accounted through figures and tables. Moreover, the heat and mass transfer rates also reflected with variations of the governing parameters. The exactness of present computations also verified through a comparison of prior published results and found to be in great consent.
KEYWORDS; -*Double dispersion effect, Natural convection, Soret and Dufour effects, Chebyshev spectral quasi-linearization method.*

I. INTRODUCTION

The heat and mass transfer on porous medium saturated bodies are rising interest during the last several decennaries due to its broad experimental applications in current industries, for instance, building components design for energy consideration, solar power collectors, compact heat exchangers, and food industries. In the last several year's numbers of analyses has been made on natural convection adjacent to heated geometries saturated porous medium, over the wavy surface [1], in a circular cylinder [2] and cavities [3].

Hossain et al. [4] studied surface heat flux and uniform surface temperature on natural convection flow along a vertical circular cone in a thermally stratified medium. Pop and Yen [5] proposed a natural convection study adjacent to a wavy cone, Srinivasacharya and Upendar [6] introduced the impact of Soret and Dufour effects. The study of double dispersion effect with Lie group scaling transformation produced by Pranitha et al. [7] and some more fascinating work over cone can be considered as [8]-[10].

The study considered in this article has not analyzed by any researcher so far which we aimed through this article. The appropriate computations are reflected in their proper applications. The exactness of the present results also verified through a comparison with published ones.

II. MATHEMATICAL FORMULATION

Considers a vertical cone with half-angle ω saturated porous medium in the presence of double dispersion, and Soret and Dufour effects together. The vertex of the cone is fixed at the origin of the coordinate system, x axis is along the surface of the cone and y is normal, as displayed in Fig. The surface of the cone is maintained at a constant temperature T_w. C_w is concentration which is varying to C_∞. The governing equations of the model are given as:

$$\frac{\partial(\mathrm{r}u)}{\partial x}+\frac{\partial(\mathrm{r}\,v)}{\partial y}=0 \qquad (1)$$

$$u=\frac{gK\cos\omega}{\upsilon}[\beta_T(\mathrm{T}-\mathrm{T}_\infty)+\beta_C(C-C_\infty)] \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right)+\overline{D}\frac{\partial^2 C}{\partial y^2} \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_e\frac{\partial C}{\partial y}\right)+\overline{S}\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The boundary conditions are:

$$v=0,\ T=T_w,\ C=C_w \text{ at } y=0$$

$$u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty \qquad (5)$$

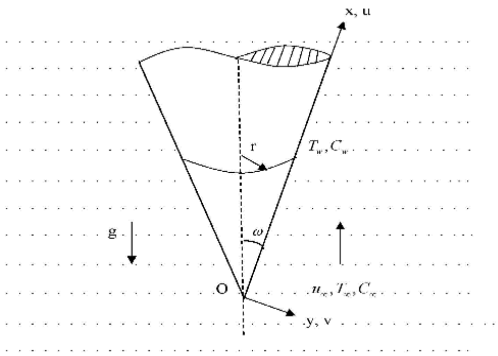

Fig. Physical geometry and coordinates

Here, the local radius of the cone is $r=x\sin\omega$, the effective thermal and Solutal diffusivities are α_e and D_e which can be written as $\alpha_e=\alpha_m+\gamma du$ and $D_e=D_m+\zeta du$, where α_m and D_m are the molecular thermal and Solutal diffusivities, respectively. $\overline{S}$ and $\overline{D}$ are the Soret and Dufour coefficients of the porous medium, respectively.

We propose the similarity variables:

$$\eta=\frac{y}{x}Ra_x^{1/2},\ f(\eta)=\frac{\psi}{\alpha r}Ra_x^{-1/2},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \qquad (6)$$

The stream function ψ is defined as:

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y},\ v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \qquad (7)$$

Enforcing Eqs. (6) and (7) into Eqs. (2)-(4), then got the subsequent non-dimensional forms as:

$$f'-\theta-N\phi=0 \qquad (8)$$

$$\theta''+\frac{3}{2}f\theta'+\mathrm{Pe}_\gamma\frac{Gr}{\mathrm{Re}}(\mathrm{f}''\theta'+\mathrm{f}'\theta'')+Da\phi''=0 \qquad (9)$$

$$\frac{\phi''}{Le}+\frac{3}{2}f\phi'+\mathrm{Pe}_\zeta\frac{Gr}{\mathrm{Re}}(\mathrm{f}''\phi'+\mathrm{f}'\phi'')+Sa\theta''=0 \qquad (10)$$

The boundary conditions Eq. (5) reduced as:

$$f=0,\ \theta=1,\ \phi=1 \text{ at } \eta=0$$

$$f' \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } \eta \to \infty \tag{11}$$

Where primes are denoting derivatives with respect to η, $N = \dfrac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$ is the buoyancy ratio, $Ra_x = \dfrac{gK\beta_T\, x(T_w - T_\infty)\cos\omega}{\upsilon\alpha} = Gr\,\mathrm{Pr}$ is the Rayleigh number, $\mathrm{Re} = \dfrac{u_\infty x}{\nu}$ is the Reynolds number, $Pe_\gamma = \dfrac{\gamma d u_\infty}{\alpha_m}$ is the thermal dispersion parameter, $Sa = \dfrac{\bar{S}(T_w - T_\infty)}{\alpha_m (C_w - C_\infty)}$ is the Soret parameter, $Pe_\zeta = \dfrac{\zeta d u_\infty}{D_m}$ is the Solutal dispersion parameter, $Le = \dfrac{\alpha_m}{D_m}$ is the Lewis number and $Da = \dfrac{\bar{D}(C_w - C_\infty)}{\alpha_m (T_w - T_\infty)}$ is the Dufour parameter.

The non-dimensional form of local Nusselt and Sherwood number can be noted as:

$$\frac{Nu_x}{Ra_x^{1/2}} = -(1 + \mathrm{Pe}_\gamma \mathrm{f}'(0))\theta'(0) \text{ and } \frac{Sh_x}{Ra_x^{1/2}} = -(1 + \mathrm{Pe}_\zeta \mathrm{f}'(0))\phi'(0) \tag{12}$$

III. NUMERICAL SOLUTION

To compute the numerical computations here, we have considered a novel numerical approach called as Chebyshev spectral quasi-linearization method (SQLM) for solving the similarity Eqs. (08)-(10) with subjective boundary conditions Eqn. (11). The linearize forms of Eqs. (08)-(10) using the quasi-linearization method (QLM) (for brief see Bellman and Kalaba [11]) are given as:

$$f'_{r+1} - \theta_{r+1} - N\phi_{r+1} = R_{1,r} \tag{13}$$

$$k_{0,r} f''_{r+1} + k_{1,r} f'_{r+1} + k_{2,r} f_{r+1} + k_{3,r}\theta''_{r+1} + k_{4,r}\theta'_{r+1} + k_{5,r}\phi''_{r+1} = R_{2,r} \tag{14}$$

$$l_{0,r} f''_{r+1} + l_{1,r} f'_{r+1} + l_{2,r} f_{r+1} + l_{3,r}\theta''_{r+1} + l_{4,r}\phi''_{r+1} + l_{5,r}\phi'_{r+1} = R_{3,r} \tag{15}$$

Where $k_{i,r}$, (i = 0,1,2,..,5) and $l_{j,r}$, (j = 0,1,2,..,5) are the coefficients such as:

$$k_{0,r} = \frac{Gr}{\mathrm{Re}} Pe_\gamma \theta'_r,\quad k_{1,r} = \frac{Gr}{\mathrm{Re}} Pe_\gamma \theta''_r,\quad k_{2,r} = \frac{3}{2}\theta'_r,\quad k_{3,r} = 1 + \frac{Gr}{\mathrm{Re}} Pe_\gamma f'_r,\quad k_{4,r} = \frac{3}{2} f_r + \frac{Gr}{\mathrm{Re}} Pe_\gamma f''_r,$$

$$k_{5,r} = Da,\quad l_{0,r} = \frac{Gr}{\mathrm{Re}} Pe_\zeta \phi'_r,\quad l_{1,r} = \frac{Gr}{\mathrm{Re}} Pe_\zeta \phi''_r,\quad l_{2,r} = \frac{3}{2}\phi'_r,\quad l_{3,r} = Sa,\quad l_{4,r} = \frac{1}{Le} + \frac{Gr}{\mathrm{Re}} Pe_\zeta f'_r,$$

$$l_{5,r} = \frac{3}{2} f_r + \frac{Gr}{\mathrm{Re}} Pe_\zeta f''_r,$$

$$R_{1,r} = 0,\ R_{2,r} = \frac{3}{2} f_r\theta'_r + Pe_\gamma \frac{Gr}{\mathrm{Re}}(f''_r\theta'_r + f'_r\theta''_r),\ R_{3,r} = \frac{3}{2} f_r\phi'_r + Pe_\zeta \frac{Gr}{\mathrm{Re}}(f''_r\phi'_r + f'_r\phi''_r)$$

The boundary conditions are:

$f_{r+1}(\eta)=0, \theta_{r+1}(\eta)=1, \phi_{r+1}(\eta)=1$ at $\eta=0$

$$f'_{r+1}(\eta)=0, \theta_{r+1}(\eta)=0, \phi_{r+1}(\eta)=0 \text{ as } \eta\to\infty \tag{16}$$

The obtained iterative linear system of Eqs. (13)-(15) by QLM with above defined variable coefficients can be solved by any numerical technique for $r=1,2,3,...$ Here, we have considered the Chebyshev pseudo-spectral method (briefly see Meena [12]). Considered Gauss-Lobatto kind collocation points are defined as:

$$\xi_j=\cos\left(\frac{j\pi}{N}\right),\ j=0,1,2,...,N \tag{17}$$

Where N is representing number of collocation points which we have considered 200 here.

The matrix system of Eqs. (13)-(15) is noted as:

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}\begin{bmatrix} f_{r+1} \\ \theta_{r+1} \\ \phi_{r+1} \end{bmatrix}=\begin{bmatrix} R_{1,r} \\ R_{2,r} \\ R_{3,r} \end{bmatrix} \tag{18}$$

Where, $M_{11}=D,\ M_{12}=-I,\ M_{13}=-NI,\ M_{21}=k_{0,r}D^2+k_{1,r}D+k_{2,r}I,\ M_{22}=k_{3,r}D^2+k_{4,r}D,$

$M_{23}=k_{5,r}D^2,\ M_{31}=l_{0,r}D^2+l_{1,r}D+l_{2,r}I,\ M_{32}=l_{3,r}D^2,\ M_{33}=l_{4,r}D^2+l_{5,r}D.$

In the above expressions, $k_{i,r},(\mathrm{i}=0,1,2,..,5)$ and $l_{j,r},(j=0,1,2,3)$ are diagonal matrices, I is the identity matrix, $M_{ij},(\mathrm{i,j}=1,2,3)$ are matrices of order $(N+1)\times(N+1)$, respectively. $f,\theta,\phi,R_{1,\mathrm{r}},R_{2,\mathrm{r}}$ and $R_{3,\mathrm{r}}$ are column matrices of order $(N+1)\times1$ and the subscript r represents the number of iteration.

IV. RESULTS AND DISCUSSION

The simplified equations (8)-(10) with boundary condition Eq. (11) are solved numerically via Chebyshev spectral quasi-linearization method (SQLM). Fig. 1 and 2 display the impact of Soret (Sa) and Dufour (Da) parameters on temperature profile, the Soret parameter reduces temperature profile but the Dufour parameter is contrary, i.e. increasing. Fig. 3 and 4 are reflecting the impact of thermal dispersion (Pe_γ) and Solutal dispersion (Pe_ζ) parameters on temperature profile, and the temperature profile enhances with Pe_γ but, contrary to Pe_ζ. Fig. 5 and 6 are presenting local Nusselt and local Sherwood numbers over Soret parameter (Sa) for variation of buoyancy ratio (N) and Lewis number (Le), Sa increases Nu_x but contrary on Sh_x i.e. decrease, Nu_x rises with N but opposite with Le. Likewise, a similar trend on Sh_x with N but contrary with Le i.e. Sh_x enhances with N and Le both. Fig. 7 and 8 are introducing local Nusselt and local Sherwood numbers over Dufour parameter (Da) for variation of buoyancy ratio (N) and

Lewis number (Le) Sh_x increases with Da but Nu_x reduces. N pushed Nu_x and Sh_x both in an upward direction. Likewise, Le rises Sh_x but reduces Nu_x.

Table 1 produces a comparison of present results with published ones Yih (1999) [8] and Cheng (2009) [13], which are in great consent. Table 2 produces the local Nusselt and local Sherwood numbers for the variation of governing parameters. The Nu_x and Sh_x both enhance with N, an increment in Le increases Sh_x and reduces Nu_x. The Sa and Pe_ζ both reduces Sh_x and contrary on Nu_x i.e. enhances. Likewise, Da and Pe_γ both reduce Nu_x and increases Sh_x, respectively.

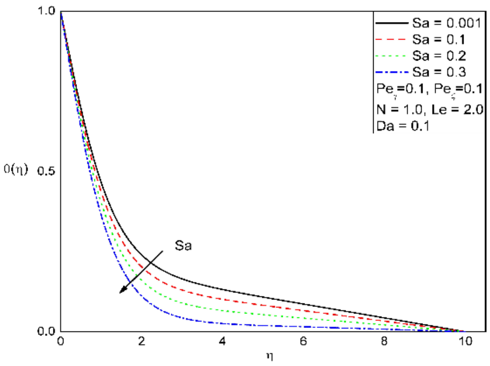

Fig. 1 Variation of Sa on temperature profile

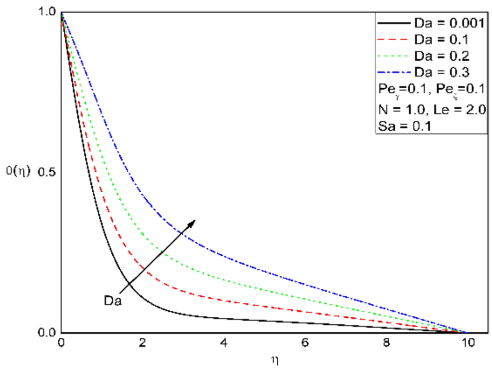

Fig. 2 Variation of Da on temperature profile

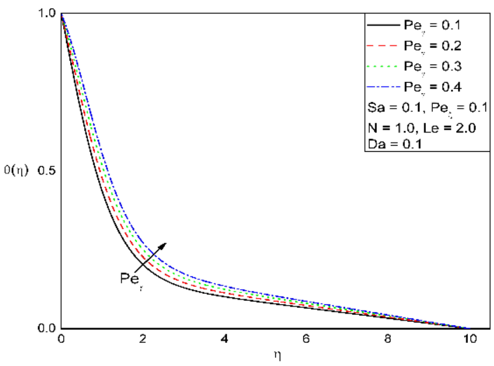

Fig. 3 Variation of Pe_γ on temperature profile

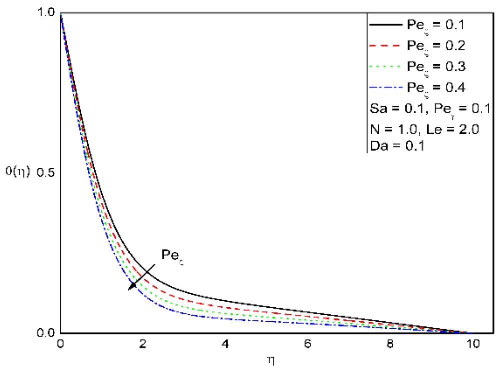

Fig. 4 Variation of Pe_ζ on temperature profile

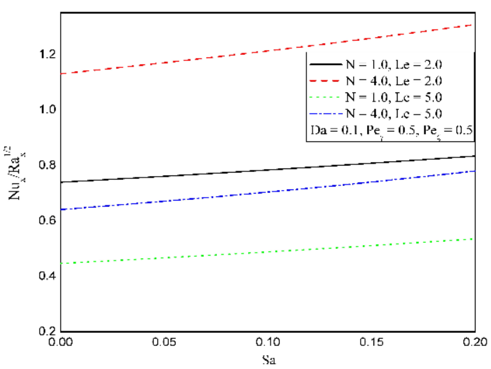

Fig. 5 Variation of Sa on local Nusselt number

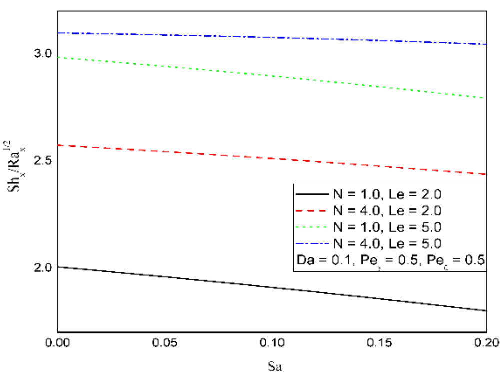

Fig. 6 Variation of Sa on local Sherwood number

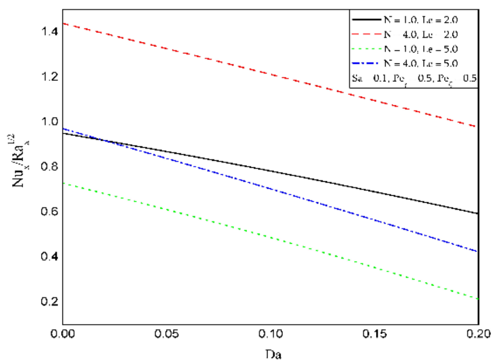

Fig. 7 Variation of Da on local Nusselt number

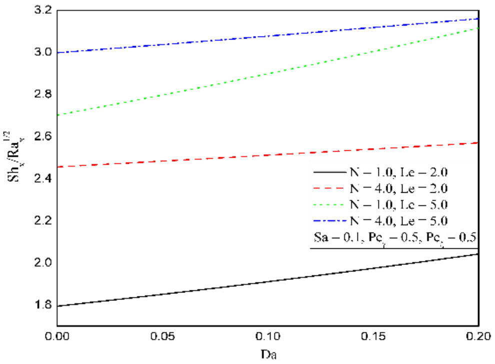

Fig. 8 Variation of Da on local Sherwood number

Table. 1. Comparison of values of $-\theta'(0)$ and $-\phi'(0)$ for $Da = Sa = Pe_\gamma = Pe_\zeta = 0, Le = 1.0$.

	$-\theta'(0)$			$-\phi'(0)$		
N	Yih (1999)	Cheng (2009)	Present	Yih (1999)	Cheng (2009)	Present
4.0	1.7186	1.7186	1.71863	1.7186	1.7186	1.71863
1.0	1.0869	1.0870	1.08685	1.0869	1.0870	1.08685
0.0	0.7686	0.7686	0.76858	0.7686	0.7686	0.76858

Table. 2. Local Nusselt and local Sherwood numbers for variation of N, Le, Sa, Da, Pe_γ and Pe_ζ .

N	Le	Sa	Da	Pe_γ	Pe_ζ	Nu_x	Sh_x
0.0	1.0	0.1	0.1	0.2	0.2	0.77912732	0.77912732
1.0						1.18585539	1.18585539
4.0						2.04185731	2.04185731
1.0	1.0					1.18585539	1.18585539
	5.0					0.41081046	2.78351123
	10.0					0.32561458	3.09376601
		0.01				1.13094111	1.27624520
		0.1				1.18585539	1.18585539
		0.2				1.25449696	1.07314670
		0.3				1.33286656	0.94481403
			0.01			1.27624520	1.13094111
			0.1			1.18585539	1.18585539
			0.2			1.07314670	1.25449696
			0.3			0.94481403	1.33286656
				0.1		1.19546289	1.12612754
				0.2		1.18585539	1.18585539
				0.3		1.15074992	1.24095414
				0.4		1.09505966	1.29218667
					0.1	1.12612754	1.19546289
					0.2	1.18585539	1.18585539
					0.3	1.24095414	1.15074992
					0.4	1.29218667	1.09505966

V. CONCLUSION

This article produced the impact of double dispersion in the presence of Soret and Dufour effects over a vertical cone saturated porous medium with Newtonian fluids due to natural convection, and we reached to conclude:

- As the Dufour Da and thermal dispersion parameter Pe_γ increases then Sh_x enhances but the Nu_x decreases and the similar trend took for variation of Lewis number Le also.
- An increment in Soret Sa and Solutal dispersion Pe_ζ parameters increases Nu_x but reduces Sh_x. The buoyancy ratio N increases both the local Nusselt Nu_x and Sherwood numbers Sh_x.

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